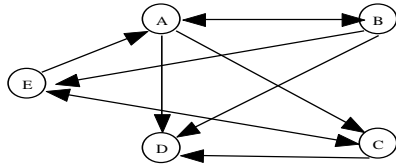


GRAPH THEORY

1. Given the directed graph with vertices A, B, C, D, and E and directed edges AB, AC, AD, BA, CD, EA, EC, CE, BD, and BE. How many pairs of vertices have no directed edge?

2

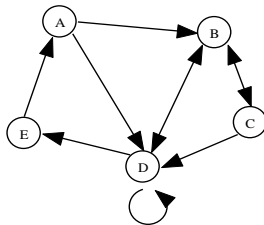
B & C and D & E



2. Given the directed graph with vertices A, B, C, and D, and directed edges AB, BD, CD, AC, CB, CA, DA, and AD. Draw the adjacency matrix represented by the directed graph.

$$\begin{vmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{vmatrix}$$

3. How many paths of length 2 exist in the following directed graph?



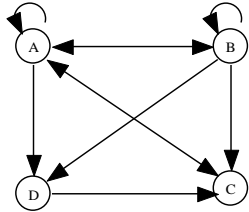
23

Squaring an adjacency matrix produces all paths of length 2. Adding the entries gives the paths of length 2.

$$\begin{vmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{vmatrix}^2 = \begin{vmatrix} 0 & 1 & 1 & 2 & 1 \\ 0 & 2 & 0 & 2 & 1 \\ 0 & 1 & 1 & 2 & 1 \end{vmatrix}$$

$$\begin{vmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{vmatrix} \quad \begin{vmatrix} 1 & 1 & 1 & 2 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{vmatrix}$$

4. How many paths of length 3 exist in the following directed graph?



64

Cubing an adjacency matrix produces all the paths of length 3. Adding the entries gives the paths of length 3.

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{vmatrix}^3 = \begin{vmatrix} 8 & 5 & 7 & 5 \\ 8 & 5 & 7 & 5 \\ 3 & 2 & 3 & 2 \\ 1 & 1 & 1 & 1 \end{vmatrix}$$